Period 1, Sept 4, 2024

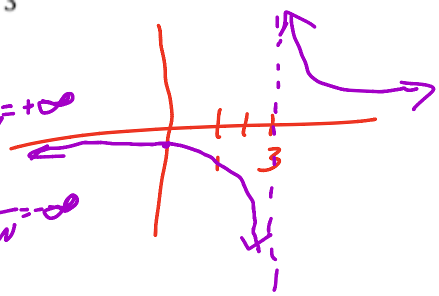
4. For each of the following, identify all vertical asymptotes and find $\lim_{x\to a^+} f(x)$, $\lim_{x\to a^-} f(x)$, $\lim_{x\to a} f(x)$ where a is the x-value of the asymptote. Show your work.

a) $f(x) = \dfrac{1}{x-3}$

X=3

$\lim_{x\to 3^+} F(x) = \dfrac{1}{+RSN} = +\infty$

$\lim_{x\to 3^-} F(x) = \dfrac{1}{-RSN} = -\infty$



b) $\cancel{f(x) = \dfrac{1}{x^2 - 4x + 4}}$

6. Sketch a function that satisfies the stated conditions. Include any asymptotes.

$\lim_{x\to 3^-} g(x) = \infty$ X=3 asy

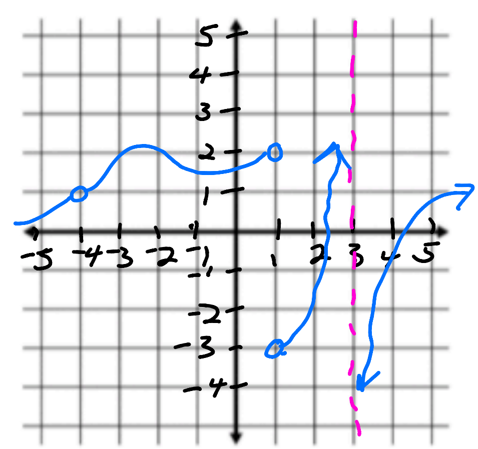
$\lim_{x\to 1^+} g(x) = -3$ (1,-3) Right

$\lim_{x\to 3^+} g(x) = -\infty$

$\lim_{x\to -4} g(x) = 1$ (-4,1) open circle

$g(x)$ is not continuous at $x = -4$

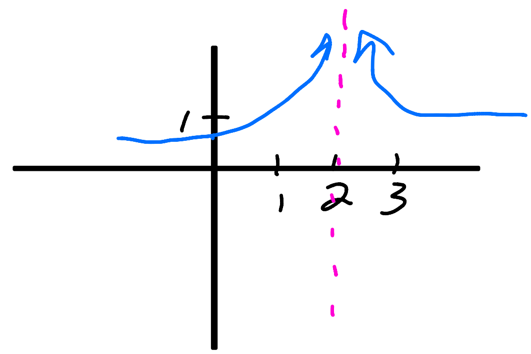
$\lim_{x\to 1^-} g(x) = 2$ (1,2) From Left



5. Find the following limits. Show your work.

a) $\lim_{x \to 5^+} \frac{3x-5}{5-x}$

b) $\lim_{x \to 2} \frac{3}{(x-2)^2} = \frac{3}{+RSN} = +\infty$



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1. Find the vertical asymptote(s) and list the removable discontinuities (if any) of
$f(t) = \frac{t(t+2)}{(t^2-4)(4t^2-2t-12)}$

Factor

$T^2 - 4 = T^2 - 2^2 = (T-2)(T+2)$

$4T^2 - 2T - 12 = 2[2T^2 - T - 6]$

$2 \cdot -6 = -12$

$-4 + 3 = -1$

$2T^2 - 4T + 3T - 6$

$2T(T-2) + 3(T-2)$

$(T-2)(2T+3)$

$F(T) = \frac{T\cancel{(T+2)}}{\cancel{(T+2)}(T-2) \cdot 2 \cdot (T-2)(2T+3)}$

$T = -2$ Removable (hole)

asymptotes $T = 2$ and $T = -\frac{3}{2}$

$\lim_{T \to 2} F(T) = +\infty$

$+\infty = \lim_{T \to -\frac{3}{2}^-} F(T) = \frac{-\frac{3}{2}}{-RSN}$

$\lim_{T \to -\frac{3}{2}^+} F(T) = \frac{-\frac{3}{2}}{+RSN} = -\infty$

$$\sin(a+b)=\sin a\cos b+\sin b\cos a$$
$$\sin 2a = 2\sin a\cos a$$
$$\sin(a-b)=\sin a\cos b-\sin b\cos a$$

$$\boxed{\lim_{a\to 0}\frac{\sin a}{a}=1=\lim_{a\to 0}\frac{a}{\sin a}}$$

17. $\lim\limits_{x\to 0}\dfrac{\sin(2x)\sin(5x)}{x\sin(3x)}=$

$$\lim_{x\to 0}\frac{5\sin 2x\cdot\sin 5x}{5x\cdot\sin 3x}$$

$$\lim_{x\to 0}\left[\frac{5\cdot\sin 2x}{\sin 3x}\right]\left[\frac{\sin 5x}{5x}\right]$$

$$\lim_{x\to 0}\frac{5\cdot x\sin 2x}{x\cdot\sin 3x}=\lim_{x\to 0}5\left[\frac{2\cdot\sin 2x}{2x}\right]\cdot\left[\frac{3x}{3\sin 3x}\right]=\frac{5\cdot 2}{3}=\frac{10}{3}$$

1. $f(x)=\dfrac{2x-1}{4x^2-3}$ $=\lim\limits_{x\to\infty}\dfrac{2x-1}{4x^2-3}=\lim\limits_{x\to\infty}\dfrac{2x}{4x^2}=\lim\limits_{x\to\infty}\dfrac{1}{2x}=\dfrac{1}{RBN}=0$

$\lim\limits_{x\to-\infty}\dfrac{2x-1}{4x^2-3}=\lim\limits_{x\to-\infty}\dfrac{1}{2x}=\dfrac{1}{-RBN}=0$

2. $g(x)=\dfrac{4x^2-3}{2x-1}$ $=\lim\limits_{x\to\infty}\dfrac{4x^2-3}{2x-1}=\lim\limits_{x\to\infty}\dfrac{4x^2}{2x}=\lim\limits_{x\to\infty}2x=\infty$ End behavior

$\lim\limits_{x\to-\infty}\dfrac{4x^2-3}{2x-1}=\lim\limits_{x\to-\infty}\dfrac{4x^2}{2x}=\lim\limits_{x\to-\infty}2x=-\infty$

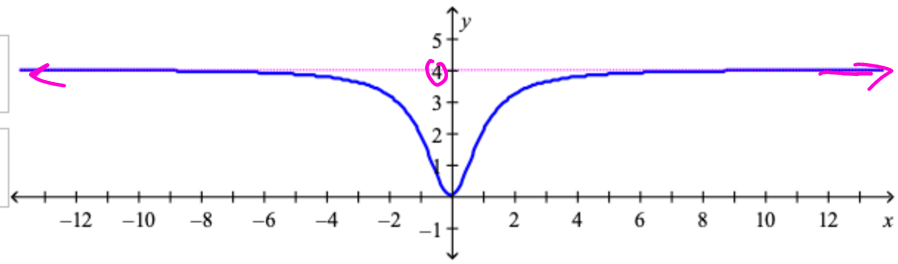
3. $h(x)=\dfrac{4x^2-2x}{4x^2+1}$ $=\lim\limits_{x\to\infty}\dfrac{4x^2-2x}{4x^2+1}=\lim\limits_{x\to\infty}\dfrac{4x^2}{4x^2}=1$

$\lim\limits_{x\to-\infty}\dfrac{4x^2-2x}{4x^2+1}=\lim\limits_{x\to\infty}\dfrac{4x^2}{4x^2}=1$

$f(x) = \frac{4x^2}{x^2+1}$

$\lim_{x\to\infty} \frac{4x^2}{x^2+1} = \boxed{4}$

$\lim_{x\to-\infty} \frac{4x^2}{x^2+1} = \boxed{4}$



$\lim_{x\to\infty} \frac{4x^2}{x^2+1} = \lim_{x\to\infty} \frac{4\cancel{x^2}}{\cancel{x^2}} = 4$

Relationship of Exponents	Horizontal Asymptote
Degree of Numerator is larger	$\lim_{x\to\infty}\frac{x^3}{x^2+1} = \lim_{x\to\infty}\frac{x^3}{x^2} = \lim_{x\to\infty} x = \infty$
Degree of Denominator is larger	$\lim_{x\to\infty}\frac{x^2+1}{x^3} = \lim_{x\to\infty}\frac{x^2}{x^3} = \lim_{x\to\infty}\frac{1}{x} = 0$
Degrees are the same	$\lim_{x\to\infty}\frac{5x^4-3{,}000000x+5}{3x^4+300{,}000{,}000x^3-1} = \lim_{x\to\infty}\frac{5x^4}{3x^4} = \frac{5}{3}$

$x = -300$

b) $\lim_{x\to-\infty} e^x - 2x$

$\frac{1}{e^{300}} - 2(-300) = \frac{1}{RBN} + 600$

e) $$\lim_{x\to\infty} \frac{\sqrt{x^2+3x+1}}{4x}$$

$$= \lim_{x\to\infty} \frac{\sqrt{x^2}}{4x} = \lim_{x\to\infty} \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}$$

$$\lim_{x\to\infty} \frac{|x|}{4x} = \lim_{x\to\infty} \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}$$

f) $$\lim_{x\to-\infty} \frac{\sqrt{x^2+3x+1}}{4x}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2}}{4x} = \frac{|x|}{4x} = \frac{+\cancel{x}}{4(-\cancel{x})} = \frac{-1}{4}$$

$$\frac{+\cancel{BBN}}{4(-\cancel{BBN})}$$